Math 3124A/9024A Assignment 4

University of Western Ontario

Fall 2023

1. (Bak–Newman E4.3) In class we integrated $1/z$ along the curve parametrized by $z(t) = \cos(t) + i\sin(t)$, $t \in [0, 2\pi]$ (see Example 2 in §4.1 of the text). This time, integrate $1/z$ along the curve C parametrized by $w(t) = \sin(t) + i\cos(t)$, $t \in [0, 2\pi]$. Why are the two answers different?

2. (Bak–Newman E4.6) Show that, if f is a continuous *real-valued* function and $|f| \leq 1$, then
$$\left| \int_C f(z)\mathrm{d}z \right| \leq 4,$$
where C is the unit circle parametrized counterclockwise. [*Hint:* Show that $\left|\int f\right| \leq \int_0^{2\pi} |\sin t|\mathrm{d}t$.]

3. (Bak–Newman E4.8) Show that $\int_C z^k \mathrm{d}z = 0$ for any integer $k \neq -1$ and $C : z = Re^{i\theta}$, $0 \leq \theta \leq 2\pi$

   a. by showing that z^k is the derivative of a function analytic throughout C,

   b. directly, using the parametrization of C.

4. (Bak–Newman E4.11) Suppose that f is analytic in a convex region D and $|f'| \leq 1$ throughout D. Prove that f is a "contraction"; i.e., show that $|f(b) - f(a)| \leq |b - a|$ for all $a, b \in D$.

5. **[MATH 9024 STUDENTS ONLY]** (Bak–Newman E4.12) Let a, b be two numbers in the left half-plane. Prove that $|e^a - e^b| < |a - b|$.