Mathematics 3120B **Assignment 1** 26 January 2024

Please upload your assignment onto Gradescope.ca before 10:00AM on the due date shown above. No late assignments will be accepted. Write all answers in complete sentences. You must do your own work.

1. Let G be a group, and let $x, g \in G$. Prove that $|x| = |g^{-1}xg|$. Deduce that $|ab| = |ba|$, for all $a, b \in G$.

2. Label the vertices of a rectangle that is not a square by 1, 2, 3, 4. Represent its symmetry group K as a subgroup of S_4. What is the order of each element in K?

3. How many elements are there of each possible order in S_6?

4. Prove that D_{24} and S_4 are not isomorphic.

5. Let G be a group. Prove that the map $\varphi : G \to G : g \mapsto g^2$ is a homomorphism if and only if G is Abelian.

6. Let G be group. Show that the map $G \times G \to G$ given by $g \cdot a = ag^{-1}$, for all $a, g \in G$ defines a (left) action of G on itself.

7. Let G be a group and suppose that G acts on a set A. Prove that the relation on A given by $a \sim b$ if and only if $a = g \cdot b$, for some $g \in G$, is an equivalence relation.

8. Let $G = S_n$, fix $i \in \{1, 2, \ldots, n\}$, and let $G_i = \{\sigma \in G \,|\, \sigma(i) = i\}$. Use group actions to prove that G_i is a subgroup of G. Find $|G_i|$.