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HW 1

Math 3122
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1. *Proof.* Suppose for the sake of contradiction that such a function f existed. Then f is differentiable so it is continuous. By the MVT, for some $0 < c < x$, $\frac{f(x)-f(0)}{x-0} = f'(c) = 1$ when $x > 0$. So, $\lim_{x\to 0^+} \frac{f(x)-f(0)}{x} = 1 \neq 0$. Thus, $\lim_{x\to 0} \frac{f(x)-f(0)}{x-0} \neq 0$, so $f'(0) \neq 0$, contradicting the supposition. □

2. *Proof.* Let $F(x) = \int_a^x f(x)dx$. By the Fundamental Theorem of Calculus 2, $F'(x) = f(x)$ and F is continuous. By the MVT, $\frac{F(b)-F(a)}{b-a} = F'(c) = f(c)$ for some $c \in (a, b)$. That is, $F(b) - F(a) = F(b) = \int_a^b f(x)dx = f(c)(b-a)$. □