MATH 3122A FALL 2023 HOMEWORK ASSIGNMENT 4

Please hand in your solutions to T. Barron on or before Wed. November 22 (10:30 am)

Problem 1.
Recall that if S is a nonempty subset of a metric space (X, d_X), we can restrict the metric

$$d_X : X \times X \to \mathbb{R}$$

to a function $S \times S \to \mathbb{R}$ and thus obtain an induced metric on S.

Let A and B be subsets of a metric space (X, d_X) such that $A \cap B \neq \emptyset$. Prove: if A and B are connected, then $A \cup B$ is connected.

Problem 2. Let (X, d_X), (Y, d_Y) be metric spaces. Suppose $f : X \to Y$ is a homeomorphism. Prove: X is connected if and only if Y is connected.